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First order chemical reaction effects on exponentially accelerated isothermal vertical plate with variable mass diffusion in the presence of radiation

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ABSTRACT

Closed form solution of thermal radiation effects on, unsteady free convective flow of a viscous incompressible fluid, past an exponentially accelerted infinite isothermal vertical plate with variable mass diffusion, in the presence of homogeneous chemical reaction of first order has been considered. The plate temperature is increased and the concentration level near the plate is raised lineraly with time. An exact solution to the dimensionless governing equations are solved by the Laplace transform method, when the plate is exponentially accelerated in its own plane against the gravitational field. The effects of velocity, temperature and concentration are studied for different physical parameters like thermal radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing values of the chemical reaction parameter or radiation parameter. But the trend is just reversed with respect to parameter time.

The governing equations together with magnetohydodynamic equations are used to focus on the study of ferrofluids, interaction between fluid particles in the bloodstream and external magnetic field, which has great importance in cancer research in developing methods for delivery of medicine in affected areas

KEY WORDS: chemical reaction, radiation, isothermal, vertical plate, mass transfer.

1. INTRODUCTION

The Effect of a chemical reaction depends on whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, if it takes place at an interface, the reaction is heterogeneous; and homogeneous if it takes place in solution. (Chambre, 1958) have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. (Das, 1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by (Das, 1996).

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles. (England, 1969) have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate was studied by (Hossain, 1996). (Deka, 1999) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

(Gupta, 1979) studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. (Kafousias, 1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by (Singh, 1984). The skin friction for accelerated vertical plate has been studied analytically by (Shayo, 1986). (Basanth Kumar, 1991) analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

However the study of thermal radiation effects on unsteady flow past a exponentially accelerated isothermal vertical plate in the presence of chemical reaction of first order has not been studied in the literature. It is proposed to study the chemical reaction effects on unsteady flow past an exponentially accelerated isothermal vertical plate with variable mass diffusion, in the presence of thermal radiation. Such a study found useful in energy storage, food processing, freezing, magnetic drug targeting.

1.1. Basic Equations and Analysis

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration.At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration level C'_w near the plate is also raised linearly with respect to time. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. By usual Boussinesq's approximation, the unsteady flow is governed by the following:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l(C' - C'_\infty) \tag{3}$$

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. If the reaction rate if proportional to the n^{th} power of the concentration then the reaction is said to be of the order n. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself. The negative sign in last term of (3) indicates that the reaction takes place normally from higher concentration to lower concentration.
With the following initial and boundary conditions:

$$\begin{array}{lllll} t' \le 0: & u = 0, & T = T_\infty, & C' = C'_\infty & for\ all\ \ y \\ t' > 0: & u = u_0 \exp(a't'), & T = T_W, & C' = C'_\infty + (C'_W - C'_\infty)\,A\,t' & at\ \ y = 0 \\ & u = 0, & T \to T_\infty, & C' \to C'_\infty & as\ \ y \to \infty, \quad where\ A = \frac{u_0^2}{\nu} \end{array} \tag{4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma\left(T_\infty^4 - T^4\right) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\,T - 3T_\infty^4 \tag{6}$$

By using (5) and (6), (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3\,(T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$\begin{gathered} U = \frac{u}{u_0}, \quad t = \frac{t'u_0^2}{\nu}, \quad Y = \frac{yu_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_W - T_\infty}, \\ Gr = \frac{g\beta\nu(T_W - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_W - C'_\infty}, \quad Gc = \frac{\nu g\beta^*(C'_W - C'_\infty)}{u_0^3}, \\ R = \frac{16a^*\nu^2\sigma T_\infty^3}{ku_0^2}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}, \quad K = \frac{\nu K_l}{u_0^2}, \quad a = \frac{a'\nu}{u_0^2} \end{gathered} \tag{8}$$

in (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\,\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - K\,C \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0, & for\ all & Y, t \le 0 \\ t > 0: & U = \exp(at), & \theta = 1, & C = t, & at & Y = 0 \\ & U = 0, & \theta \to 0, & C \to 0 & as & Y \to \infty \end{array} \tag{12}$$

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All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. Equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{at})\right] \tag{13}$$

$$\begin{aligned} C = {} & \frac{t}{2}\left[\exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \\ & - \frac{\eta\sqrt{Sct}}{2\sqrt{K}}\left[\exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt}) - \exp(2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} + \sqrt{Kt})\right] \end{aligned} \tag{14}$$

$$\begin{aligned} U = {} & \frac{\exp(at)}{2}\left[\exp(2\eta\sqrt{at})\, erfc(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at})\, erfc(\eta - \sqrt{at})\right] \\ & - e\exp(ct)\left[\exp(2\eta\sqrt{ct})\, erfc(\eta + \sqrt{ct}) + \exp(-2\eta\sqrt{ct})\, erfc(\eta - \sqrt{ct})\right] \\ & - f\exp(dt)\left[\exp(2\eta\sqrt{dt})\, erfc(\eta + \sqrt{dt}) + \exp(-2\eta\sqrt{dt})\, erfc(\eta - \sqrt{dt})\right] \\ & + (e + f)erfc(\eta) + 2dt\left[(1 + 2\eta^2)erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] \\ & - e\left[\exp\left(2\eta\sqrt{Rt}\right).erfc\left(\eta\sqrt{\Pr} + \sqrt{at}\right) + \exp\left(-2\eta\sqrt{Rt}\right).erfc\left(\eta\sqrt{\Pr} - \sqrt{at}\right)\right] \\ & + e\exp(ct)\left[\exp\left(-2\eta\sqrt{\Pr(b+c)t}\right).erfc\left(\eta\sqrt{\Pr} - \sqrt{(b+c)t}\right)\right. \\ & \qquad \left. + \exp\left(2\eta\sqrt{\Pr(b+c)t}\right).erfc\left(\eta\sqrt{\Pr} + \sqrt{(b+c)t}\right)\right] \\ & - f(1+dt)\left[\exp\left(2\eta\sqrt{KtSc}\right).\, erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{KtSc}\right).\, erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \\ & + \frac{fd\eta\sqrt{Sc}\sqrt{t}}{\sqrt{K}}\left[\exp\left(-2\eta\sqrt{KtSc}\right) erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) - \exp\left(2\eta\sqrt{KtSc}\right) erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right] \\ & + f\exp(dt)\left[\exp\left(-2\eta\sqrt{Sc(K+d)t}\right).erfc\left(\eta\sqrt{Sc} - \sqrt{(K+d)t}\right)\right. \\ & \qquad \left. + \exp\left(2\eta\sqrt{Sc(K+d)t}\right).erfc\left(\eta\sqrt{Sc} + \sqrt{(K+d)t}\right)\right] \end{aligned} \tag{15}$$

where, $b = \frac{R}{Pr}, c = \frac{R}{1-Pr}, d = \frac{K\,Sc}{1-Sc}, e = \frac{Gr}{2c(1-Pr)}$, $f = \frac{Gc}{2d^2(1-Sc)}$. and $\eta = Y/2\sqrt{t}$

where erfc is called complementary error function.

2. RESULTS AND DISCUSSION

The numerical values of the velocity, temperature and concentartion are computed for different physical parameters like thermal radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr are chosen such that they represent air ($Pr = 0.71$). The purpose of the calculations given here is to assess the effects of the parameters a, R, K, Gr, Gc and Sc upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

The temperature profiles for different values of thermal radiation parameter

($R = 2, 5, 7, 10$), in the presence of air at time $t = 0.4$ are shown in figure.1. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter. This shows there is a drop in temperature due to higher thermal radiation.

Figure.2, demonstrates the effect of concentration profiles for different values of time $(t = 0.2, 0.4, 0.6, 0.8, 1)$ and $K = 0.2$ are shown in figure.2. It is observed that the wall concentartion increases with increasing values of t. The concentration profiles for different values of the Schmidt number $(Sc = 0.16, 0.3, 0.6, 2.01)$ and $K = 2$ at time $t = 0.4$ are shown in figure.3. The effect of Schmidt number is important in concentration field. As expected, the concentration increases with decreasing values of the Schmidt number. The numerical values of the Schmidt number and the corresponding species are listed in the following table.

Table.1. Numerical values

Species	Schmidt number	Name of the chemical species
H_2	0.16	Hydrogen
He	0.3	Helium
H_2O	0.6	Water Vapor
$C_6H_5CH_2CH_3$	2.01	Ethyl Benzene

Figure 4, illustrates the effect of the concentration profiles for different values of the chemical reaction parameter $(K = 2, 5, 10)$ at $t = 0.2$. The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotonic fashion from the surface to a zero value far away in the free stream. It is observed that the concentartion increases with decreasing chemical reaction parameter.

The effect of the velocity for different values of the radiation parameter $R = (0.2, 5, 10)$ $a = 1, K = 0.2, Gr = Gc = 5$ and $t = 0.4$ are shown in figure 5. The trend shows that the velocity increases with decreasing radiation parameter. Figure 6, illustrates the effect of the velocity for different values of the chemical reaction parameter $K = (0.2, 8, 20)$, $R = 5, Gr = 5, Gc = 10, a = 1$ and $t = 0.6$. The trend shows that the velocity increases with decreasing chemical reaction parameter. It is observed that the velocity decreases in the presence of high thermal radiation. The velocity profiles for different time $t = (0.2, 0.3, 0.4)$, $R = 0.2, a = 1, Gr = Gc = 5$ and $K = 5$ are shown in Figure 7. This shows that the velocity increases gradually with respect to time t.

Figure 8 demonstrates the effect of the velocity profiles for different values of thermal Grashof number $(Gr = 2,5)$ and mass Grashof number $(Gc = 2,10)$, K=5, a=1, R=0.2 and $t = 0.6$. It is clear that the velocity increases with increasing thermal Grashof number or mass Garshof number. The velocity profiles for different (*a* = 0.2 ,0.5 ,0.9 *)* Gr = Gc = 5 and R=0.2 at *t* = 0.2 are studied and presented in figure 9. It is observed that the velocity increases with increasing values of a.

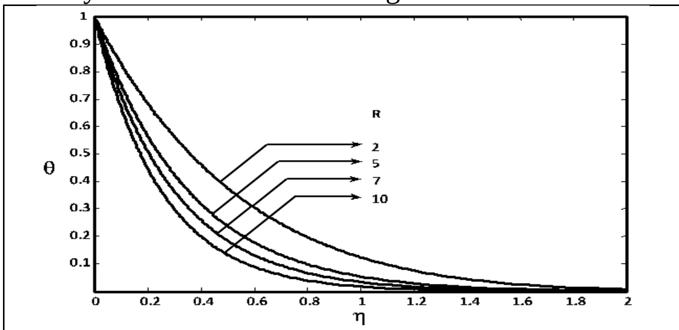

Figure.1.Temperature profiles for different values of R

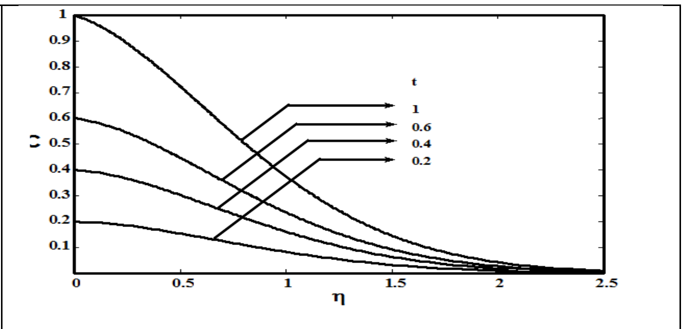

Figure.2.Concentration profiles for different values of t

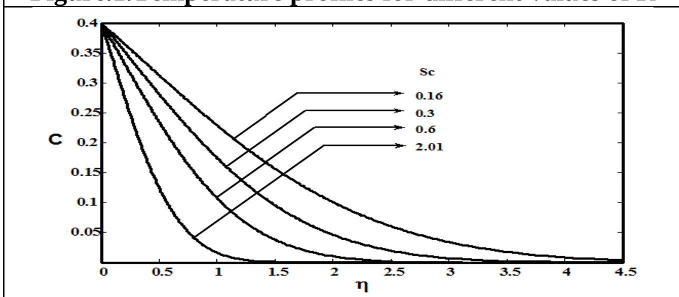

Figure.3.Concentration profiles for different values of Sc

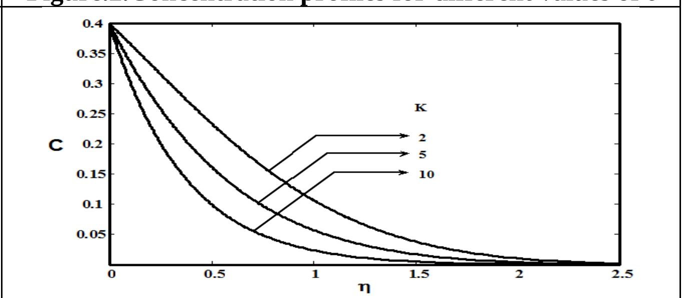

Figure.4.Concentration profiles for different values of K

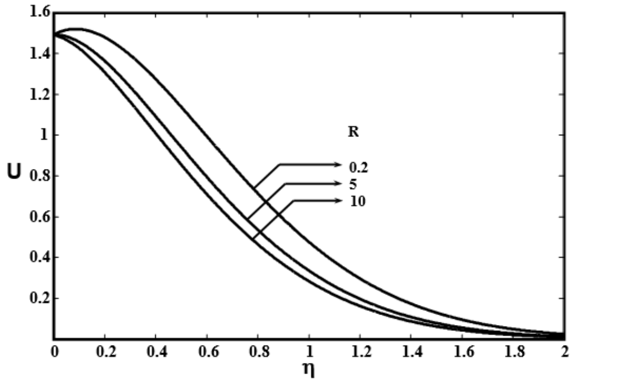

Figure.5.Velocity profiles for different values of R

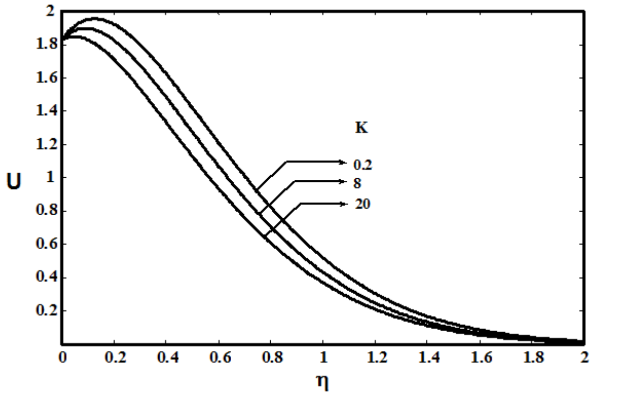

Figure 6. Velocity profiles for different values of K

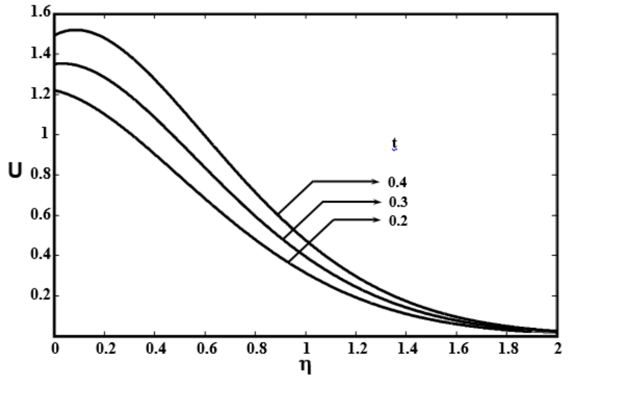

Figure.7.Velocity profiles for different values of t

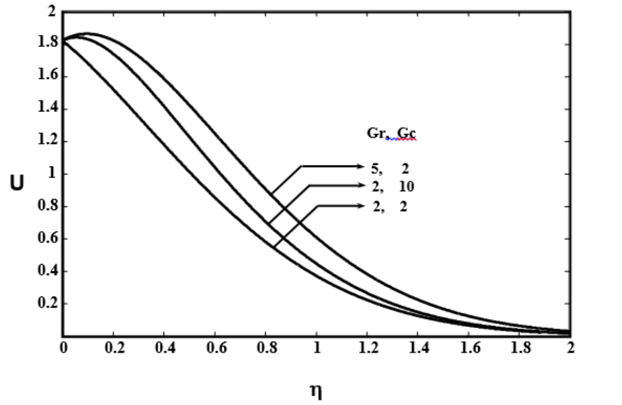

Figure.8.Velocity profiles for different values of Gr, Gc

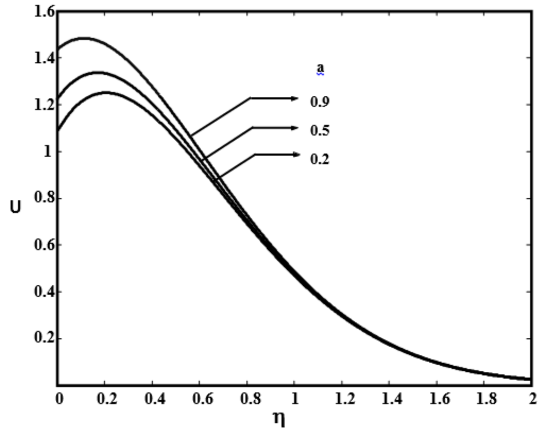

Figure.9.Velocity profiles for different values of a

3. CONCLUSION

An exact analysis of thermal radiation effects on unsteady flow past a exponentially accelerated infinite isothermal vertical plate with variable mass diffusion, in the presence of chemical reaction of first order has been studied. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like a, R, K, Gr, Gc, Sc and t are studied.We conclude that the velocity increases with decreasing chemical reaction parameter K or thermal radiation parameter R. As expected, the plate concentration increases with decreasing chemical reaction parameter and the trend is just reversed with respect to time t.

Nomenclature:

a, A, a^*	Constants	θ	Dimensionless temperature
C_p	specific heat at constant pressure	t'	time
D	mass diffusion coefficient	u	velocity of the fluid in the x-direction
Gc	mass Grashof number	u_0	velocity of the plate
Gr	thermal Grashof number	u	dimensionless velocity
g	acceleration due to gravity	y	coordinate axis normal to the plate
K	thermal conductivity	Y	dimensionless coordinate axis normal to the plate
Pr	Prandtl number	**Greek symbols**	
Sc	Schmidt number	β	volumetric coefficient of thermal expansion
T	temperature of the fluid near the plate	β^*	volumetric coefficient of expansion with concentration
C'_W	Concentration of the plate	μ	coefficient of viscosity
C'_∞	Concentration of the fluid far away from the plate	ν	kinematic viscosity
C	Dimensionless concentration	ρ	density of the fluid
T	Temperature of the fluid near the plate	τ	dimensionless skin-friction
T_W	Temperature of the plate	η	similarity parameter
T_∞	Temperature of the fluid far away from the plate	$erfc$	complementary error function

REFERENCES

Basanth Kumar Jha, Ravindra Prasad and Durendra Rai, Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux, Astrophysics and Space Science, 181, 1991, 125-134.

Chambre P.L, and Young J.D, On the diffusion of a chemically reactive species in a laminar boundary layer flow, The Physics of Fluids, 1, 1958, 48-54.

Das U.N., Deka R. and Soundalgekar V.M., Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forschung im Ingenieurwesen, 60, 1994, 284-287.

Das U.N., Deka R. and Soundalgekar V.M., Effects of mass transfer on flow past an impulsively started infinite vertical plate with chemical reaction, The Bulletin, GUMA, 5, 1999, 13-20.

Das U.N., Deka R. and Soundalgekar V.M., Radiation effects on flow past an impulsively started vertical infinite plate, J Theo. Mech., 1, 1996, 111-115.

England W.G. and Emery A.F., Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas, J Heat Transfer, 91, 1969, 37-44.

Gupta A.S., Pop I. and Soundalgekar V.M., Convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid, Rev. Roum. Sci. Techn.-Mec. Apl., 24, 1979, 561-568.

Hossain M.A. and Shayo L.K., The skin friction in the unsteady free convection flow past an accelerated plate, Astrophysics and Space Science, 125, 1986, 315-324.

Hossain M.A. and Takhar H.S., Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and Mass Transfer, 31, 1996, 243-248.

Kafousias N.G., and Raptis A.A., Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or Injection, Rev.Roum. Sci. Techn.-Mec. Apl., 26, 1981, 11-2.

Singh A.K. and Naveen Kumar, Free convection flow past an exponentially accelerated vertical plate, Astrophysics and Space Science, 98, 1984, 245-258.